Data Driven Approximate Reasoning about Changes

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Abstract. We consider several issues related to reasoning about changes starting from sensory data. In particular, we discuss challenging problems of reasoning about changes in hierarchical modeling and approximation of transition functions. This paper can be treated as a step toward developing rough calculus.

Keywords: rough sets, reasoning about changes, hierarchical modeling, granular computing, approximation space, relation (function) approximation, rough calculus, intelligent systems

1 Introduction

Reasoning about changes is one of the challenging issues in AI since the beginning of AI. In this paper, we consider a bottom up approach. We start from sensory information systems (sensory data tables) in which are recorded sensory measurements in different moments of time. Next, by using hierarchical modeling are constructed new information systems with more compound structural granules (sets of objects) such as time windows or sequences of time windows. On different levels of hierarchical modeling one can consider relations of changes, e.g., between successive time windows. Note that information systems represent only partial information about the universe of possible objects (i.e., some samples of possible objects) and the relations of changes should be induced (approximated) from partial information about the relation. We propose to use Boolean reasoning in searching for models of approximated relations. In particular, the proposed approach can be used for approximation of transition relations. Moreover, we illustrate how the approach can be extended for inducing approximation of trajectories defined by transition relations. This paper is a continuation and an extension of [5, 6]. One can also consider approximation of changes of functions relative to changes of granules representing their arguments by using the rough-set based methods (see, e.g., [12, 13]).

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2 Approximation of changes in hierarchical modeling

In this section, we start from an illustrative example of our approach to approximation of function changes. The approach is based on Boolean reasoning [4]. Next, we add some comments on approximation of changes in hierarchical modeling.

Example 1. We consider a set $U = \{x_1, \ldots, x_{12}\}$ of twelve plums observed in two time moments t_1 and t_2, where $t_1 < t_2$. We also consider a function f assigning to every object from U value "1" if and only if this object is ripe and "0" otherwise. An attribute a means hardness of objects with three possible values $l - low$, $m - middle$ and $h - high$. An attribute b is a color of plum with three possible values $g - green$, $y - yellow$ and $v - violet$. An attribute c is a size of plum with three possible values $s - small$, $m - middle$ and $l - large$.

More formally, we consider three data tables $(U^{t_i}, A^{t_i} \cup \{f^{t_i}\})$ where $i = 1, 2$ and $(\Delta U, \Delta A \cup \{\Delta f\})$ such that $U = \{x_1, \ldots, x_{12}\}$, $A = \{a, b, c\}$, $V_a = \{l, m, h\}$, $V_b = \{v, g, y\}$, $V_c = \{s, m, l\}$ and $V_f = \{1, 0\}$ (see Table 1). We define $\Delta a(x) = a^{t_1}(x) \to a^{t_2}(x)$, $\Delta b(x) = b^{t_1}(x) \to b^{t_2}(x)$, $\Delta c(x) = c^{t_1}(x) \to c^{t_2}(x)$ and $\Delta f(x) = f^{t_1}(x) \to f^{t_2}(x)$, where $x \in \Delta U$.

Table 1. Three data tables: tables in time moments t_1 and t_2 and table of changes Δf

U^{t_1}	a^{t_1}	b^{t_1}	c^{t_1}	f^{t_1}
$x_1^{t_1}$	l	v	l	1
$x_2^{t_1}$	l	y	l	1
$x_3^{t_1}$	m	g	m	0
$x_4^{t_1}$	m	g	m	0
$x_5^{t_1}$	m	y	m	1
$x_6^{t_1}$	m	g	m	0
$x_7^{t_1}$	m	v	m	1
$x_8^{t_1}$	h	g	s	0
$x_9^{t_1}$	h	g	s	0
$x_{10}^{t_1}$	h	v	s	0
$x_{11}^{t_1}$	h	g	s	0
$x_{12}^{t_1}$	h	y	s	0

U^{t_2}	a^{t_2}	b^{t_2}	c^{t_2}	f^{t_2}
$x_1^{t_2}$	l	v	l	1
$x_2^{t_2}$	l	y	l	1
$x_3^{t_2}$	l	g	l	1
$x_4^{t_2}$	l	g	l	0
$x_5^{t_2}$	m	y	m	1
$x_6^{t_2}$	m	g	m	0
$x_7^{t_2}$	m	v	m	1
$x_8^{t_2}$	m	y	m	0
$x_9^{t_2}$	h	y	s	1
$x_{10}^{t_2}$	h	v	s	1
$x_{11}^{t_2}$	h	g	s	0
$x_{12}^{t_2}$	h	y	s	0

ΔU	Δa	Δb	Δc	Δf
Δx_1	$l \to l$	$v \to v$	$l \to l$	$1 \to 1$
Δx_2	$l \to l$	$y \to y$	$l \to l$	$1 \to 1$
Δx_3	$m \to l$	$g \to g$	$m \to l$	$0 \to 1$
Δx_4	$m \to l$	$g \to g$	$m \to l$	$0 \to 0$
Δx_5	$m \to m$	$y \to y$	$m \to m$	$1 \to 1$
Δx_6	$m \to m$	$g \to g$	$m \to m$	$0 \to 0$
Δx_7	$m \to m$	$v \to v$	$m \to m$	$1 \to 1$
Δx_8	$h \to m$	$g \to y$	$s \to m$	$0 \to 0$
Δx_9	$h \to h$	$g \to y$	$s \to s$	$0 \to 1$
Δx_{10}	$h \to h$	$v \to v$	$s \to s$	$0 \to 1$
Δx_{11}	$h \to h$	$g \to g$	$s \to s$	$0 \to 0$
Δx_{12}	$h \to h$	$y \to y$	$s \to s$	$0 \to 0$

We compute the approximations with respect to values "1" and "0" of function f in time moment t_2. We also present the roughness coefficient.

We define $X_1 = \{x \in U : f(x) = 1\} = \{x_1, x_2, x_3, x_5, x_7, x_9, x_{10}\}$ and $X_0 = \{x \in U : f(x) = 0\} = \{x_4, x_6, x_8, x_{11}, x_{12}\}$. Let $AS_{\{a,b,c\}} = (U, IND(\{a, b, c\})$ be an approximation space and $U/IND(\{a, b, c\})$ a partition of U defined by attributes from $\{a, b, c\}$.

We obtain the lower approximation

$$LOW(AS_{\{a,b,c\}}, X_1) = \{x_1, x_2, x_7, x_{10}\},$$

and the upper approximation

$$UPP(AS_{\{a,b,c\}}, X_1) = \{x_1, x_2, x_7, x_{10}, x_3, x_4, x_5, x_8, x_9, x_{12}\}$$

and the roughness of X_1

$$R(AS_{\{a,b,c\}}, X_1) = 1 - \frac{card(LOW(AS_{\{a,b,c\}}, X_1))}{card(UPP(AS_{\{a,b,c\}}, X_1))} = 1 - 4 : 10 = 0.6.$$

In the similar way we obtain the roughness of X_0

$$R(AS_{\{a,b,c\}}, X_0) = 1 - 2 : 8 = 0.75.$$

We obtain the partition $\Delta U/IND(AS_{\{a,b,c\}})$ of ΔU as follows:

$$\{\{\Delta x_1\}, \{\Delta x_2\}, \{\Delta x_3, \Delta x_4\}, \{\Delta x_5\}, \{\Delta x_6\}, \{\Delta x_7\}, \{\Delta x_8\}, \{\Delta x_9\},$$
$$\{\Delta x_{10}\}, \{\Delta x_{11}\}, \{\Delta x_{12}\}\}.$$

We compute the approximations of change i.e. of the set

$$Change = \{\Delta x \in \Delta U : \Delta f(x) = 0 \to 1\} = \{\Delta x_3, \Delta x_9, \Delta x_{10}\} :$$

$$LOW(AS_{\{a,b,c\}}, Change) = \{\Delta x_9, \Delta x_{10}\},$$
$$UPP(AS_{\{a,b,c\}}, Change) = \{\Delta x_3, \Delta x_4, \Delta x_9, \Delta x_{10}\}.$$

Using Boolean reasoning, we obtain two decision reducts: $\{a, b\}$ and $\{b, c\}$.

Based on the first reduct we obtain the following two types of decision rules.

Rules with accuracy equal to 1 (based on the lower approximation):

if $\Delta a = h \to h$ **and** $\Delta b = g \to y$ **then** $\Delta f = 0 \to 1$ (based on object Δx_9),

if $\Delta a = h \to h$ **and** $\Delta b = v \to v$ **then** $\Delta f = 0 \to 1$ (based on object Δx_{10}),

Rule with accuracy less than 1 (based on boundary region):

if $\Delta a = m \to l$ **and** $\Delta b = g \to g$ **then** $\Delta f = 0 \to 1$ (based on objects Δx_3 and Δx_4).

In hierarchical modeling, on each level new information systems are constructed on the basis of already constructed information systems or sensory information systems [13, 16]. For example, starting from sensory information system in which sensory measurements in different moments of time are recorded one can define on the next level an information system in which objects are time windows and attributes are (time-related) properties of these windows (see Figure 1). Operations performed on information systems are defined as unions with constraints [14]. These operations are analogous to joins with constraints considered in databases. For each new constructed information system in hierarchical modeling, changes of one attribute relative to some other ones may be induced using (approximate) Boolean reasoning [4].

It is worth mentioning that quite often this searching process is more sophisticated. For example, one can discover several relational structures (e.g., corresponding to different attributes) and formulas over such structures defining

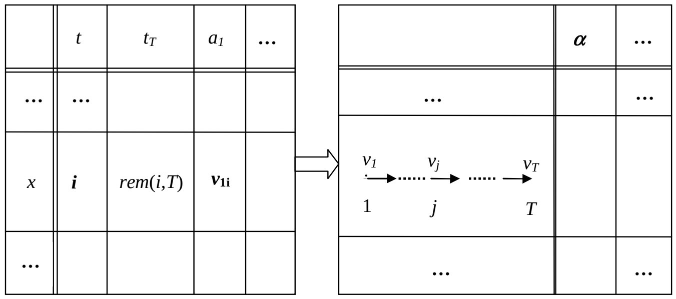

Fig. 1. Granulation of time points into time windows. A natural number $T > 0$ is the time window length, $v_j = (v_{1j}, \ldots, v_{Tj})$ for $j = 1, \ldots, T$, $rem(i, T)$ is the remainder from division of i by T, α is an attribute defined over time windows.

different families of neighborhoods from the original approximation space. As a next step, such families of neighborhoods can be merged into neighborhoods in a new, higher degree approximation space.

This approach is also relevant for Perception Based Computing [16]. For illustration let us consider an explanation of perception included in the book [1]:

> *The main idea of this book is that perceiving is a way of acting. It is something we do. Think of a blind person tap-tapping his or her way around a cluttered space, perceiving that space by touch, not all at once, but through time, by skillful probing and movement. This is or ought to be, our paradigm of what perceiving is.*

Figure 2 illustrates this idea. Note that the challenge is to discover relevant features of histories (i.e., paths of sensory measurement recordings after micro actions 'tap-tapping') for approximation of decision function whose values denote the performed actions on higher level.

3 Trajectory approximation and adaptation

One can also apply the illustrated idea of approximation of changes and Boolean reasoning for function approximation [12, 13] to transition (function) relation approximation.

First, we introduce some notation. If U^* is a set of objects and $R \subseteq U^* \times U^*$ then by XR we denote R-image of X, i.e., the set $\{y \in U^* : \ \exists x \ \in XxRy\}$. A sequence $Y_0, \ldots, Y_i, \ldots$, where $Y_0 = X$ and $Y_{i+1} = Y_i R$ for $i \geq 0$ is called R-trajectory starting at X. Let as also assume that A is a set of attributes over U^*, i.e., $a : \ U^* \longrightarrow V_a$ for any $a \in A$, where V_a is a finite set of values of the attribute a.

We consider a case when the transition relation R is partially specified by a sample, i.e. by a decision table $DT = (U_R, A \otimes A, d_R)$, where $U_R \subseteq U^* \times U^*$ is a given sample of pairs of objects, $A \otimes A = \{(a, 0) : a \in A\} \cup \{(a, 1) : a \in A\}$

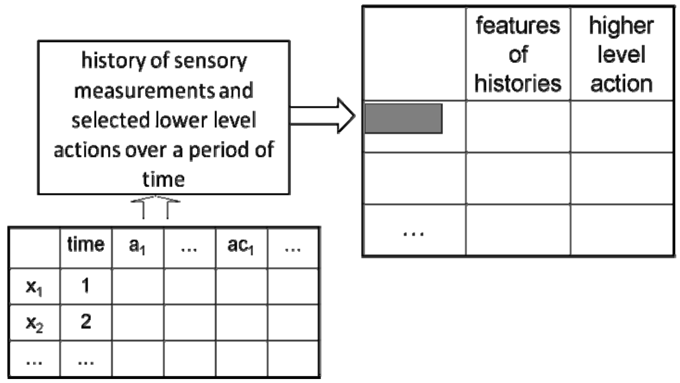

Fig. 2. Perception Idea

is the disjoint union of A (where $(a,0)(x,y) = a(x)$ and $(a,1)(x,y) = a(y)$, for $(x,y) \in U_R$), $d_R(x,y) = +$ if xRy, and $-$, otherwise, for $(x,y) \in U_R$. Hence, we assume that the transition relation is partially specified by a sample of pairs labeled by decision $+$ if the pair belongs to the relation and $-$, otherwise.

From this sample, e.g., a rule based classifier $C_A(x,y)$ may be induced [14, 13], where (x,y) is a pair of objects and $C_A(x,y) = +$ means that y is one of the next predicted states after x obtained by applying the transition relation R, and $C_A(x,y) = -$, otherwise[1].

Let us now consider a C_A-trajectory starting at the set $\|\alpha\|_{U^*}$, where α is a boolean combination of descriptors over A and $\|\alpha\|_{U^*}$ denotes the semantics of α over U^* [10].

Now, we would like to find a description of sets Y_i (for $i > 0$) in the C_A-trajectory starting at $\|\alpha\|_{U^*}$. As an illustrative example, we consider the case when α is the conjunction of descriptors from the A-signature of x [10], i.e., from $Inf_A(x) = \{(a, a(x)) : a \in A\}$ for some $x \in U^*$, and we induce the description of $\|\alpha\|_{U^*} C_A$ using boolean combinations of descriptors over A. Hence, from the elementary granule defined by $Inf_A(x)$ we derive a predicted description of its C_A-image $\|\alpha\|_{U^*} C_A$ defined by $C_A(x,y)$. We define this description by a set of attribute value vectors (more formally, by a disjunction of conjunctions of some signatures of objects).

Let us assume that $C_A(x,y)$ is a rule-based classifier based on rule set *Rule* (e.g., a subset of minimal decision rules [10]). Each rule is of the following form:

$$\textbf{if } r \textbf{ then } d = + \text{ or } \quad \textbf{if } r \textbf{ then } d = -. \tag{1}$$

The formula r can be decomposed into two parts r_1 and r_2 where r_i corresponds to the the i-th component of (x,y), where $i = 1, 2$. By $D(r_i)$ we denote the set of descriptors in r_i, where $i = 1, 2$.

[1] For simplicity of reasoning, we assume that the induced classifier takes only two values but one can extend our considerations for more values, e.g., by adding the decision value 1/2 representing borderline cases.

We restrict our considerations to the case when the decision rules of $C_A(x,y)$ are over attributes A. A more general case where the decision rules are over attributes constructed from A (see e.g., [3, 4, 15, 16, 18, 19]) will be discussed elsewhere.

Let us assume that x_0 is a new object and we would like to find the description of the image of the elementary granule defined by x_0 relative to $C_A(x,y)$. From the set *Rule* we select all rules matching x_0, i.e., all rules of the form

$$\textbf{if } r_1 \textbf{ and } r_2 \textbf{ then } d = + \text{ or } \textbf{ if } r_1 \textbf{ and } r_2 \textbf{ then } d = - \tag{2}$$

where $Inf_A(x_0)$ matches r_1.

For $v \in \{+,-\}$, we define the following sets:

$$R^v(x_0) = \{D(r_2) : \exists r_1 \ (\textbf{ if } r_1 \textbf{ and } r_2 \textbf{ then } d = v) \in \mathit{Rule} \text{ and } x_0 \text{ matches } r_1\}. \tag{3}$$

A set X of descriptors over the set of attributes A (i.e., a set of pairs (a,v), where $a \in A$ and $v \in V_a$ [10]) is consistent if the set X is a function. If X is a set of descriptors over A and $\mathcal{X}$ is a family of sets of descriptors over A then X is $\mathcal{X}$-maximal consistent if X is consistent and $X \cup Y$ is not consistent for any $Y \in \mathcal{X}$.

Example 2. A set $X = \{(\Delta a, l \to l), (\Delta b, g \to y), (\Delta c, s \to m)\}$ of descriptors over the set of attributes $\Delta A = \{\Delta a, \Delta b, \Delta c\}$ (see Table 1) is $\mathcal{X}$-maximal consistent, where $\mathcal{X} = \{\{(\Delta a, l \to l), (\Delta b, v \to v), (\Delta c, l \to l)\}, \{(\Delta a, m \to m), (\Delta b, y \to y), (\Delta c, m \to m)\}\}$.

Now, we consider all tuples (X, Y, u) such that

1. X is the union of a subset of $R^+(x_0)$;
2. Y is the union of a subset of $R^-(x_0)$;
3. $u \in INF(B) = \{(a,v) : a \in B \ \& \ v \in V_a\}$, where $B \subseteq A$ and B is disjoint with the sets of attributes occurring in $X \cup Y$;
4. $X \cup Y \cup \{u\}$ is the $(R^+(x_0) \cup R^-(x_0))$-maximal consistent set;
5. voting strategy used in construction of $C_A(x,y)$ applied to the set of all rules from *Rule* of the form (2), where r_1 is matched by x_0 and $D(r_2) \subseteq X \cup Y$ returns the decision $+$.

From the above construction it follows that the image of the elementary granule $Inf_A(x_0)$ relative to $C_A(x,y)$ can be defined as equal to the set of all extensions of $X \cup Y \cup u$, where (X, Y, u) denotes a tuple satisfying the above conditions.

Iteration of the construction presented above leads to the description by boolean combinations of descriptors of approximation of trajectory defined by $Inf_A(x_0)$ relative to the approximation $C_A(x,y)$ of the transition relation (see Figure 3).

There are several reasons explaining why the searching for approximate description of trajectories over boolean combination of descriptors may be useful. One can consider sets in the C_A-trajectories as granules. Then the granule diameter relative to the granule description by boolean combination of descriptors allows us to characterize uncertainty in identifying states (defined by signatures of objects) corresponding to this granule. The granule diameter can be easily defined if there is given the description of the granule by boolean combination of descriptors. Each such a description is equivalent to a disjunction $\alpha_1 \vee \ldots \vee \alpha_k$ of conjunctions α_i (where $1 \leq i \leq k$) of descriptors from some object signatures. Let us assume that there is given a distance function $\rho_A : INF(A) \times INF(A) \longrightarrow R_+$, where $INF(A) = \{(a, v) : \ a \in A \ \& \ v \in V_a\}$ and R_+ is the set of nonnegative reals. Then the diameter $diam_{\rho_A}(g)$ of the granule g described by the disjunction $\alpha_1 \vee \ldots \vee \alpha_k$ can be defined by $sup_{1 \leq i,j \leq k} \rho_A(u_i, u_j)$, where u_i, u_j denote sets of conjuncts occurring in α_i, α_j, respectively.

Figure 3 also illustrates the necessity of trajectory adaptation. This is caused by the fact that the approximation of the transition relation and the approximation induced from this trajectory are based on a sample of data. However, data may evolve (e.g., they are growing incrementally). Then the classifier and the trajectory approximation induced so far may no longer be of satisfactory quality starting from some moment of time. It is necessary to develop methods allowing us to measure the 'distance' between the predicted trajectory P' and the observed trajectory P. If the 'difference' is becoming not acceptable, new classifier for transition relation should be induced. This illustrates another important challenge for reasoning about changes.

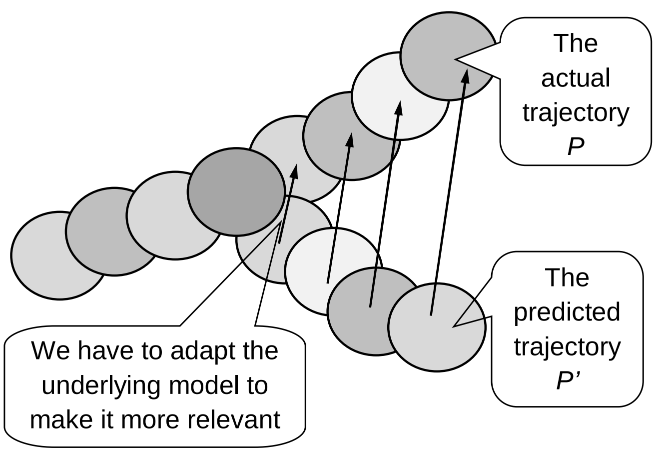

Fig. 3. Approximate trajectory adaptation

4 Rough calculus

This paper can be treated as a step toward developing rough calculus [8, 9, 2]. One possible approach to develop a concept of rough derivative is to start from a family of indiscernibility (similarity) relations defined by different choices of sensory information systems rather than a single indiscernibility relation and to characterize changes of function approximation relative to changes of indiscernibility relations. Contrary to the classical calculus, we can not expect to obtain general rules for constructing rough derivatives of $f \circ g$, where $\circ$ is a given operation on functions, from rough derivatives of f and g. However, one may induce such rules relative to given data sets (information systems).

In this section, we present an illustrative example explaining our approach. Let us discuss the concept of derivative of function in the case where the specification of the function is partial, i.e., only a sample of function is given and it is necessary to induce the function approximation. In the case of derivative, we consider a family of approximation spaces rather than a single approximation space. For simplicity of reasoning let us consider a nonincreasing chain $\{IND(A_i)\}$ of indiscernibility relations $IND(A_i)$ defined by attribute sets A_i, where $A_i \subseteq A_{i+1}$ (this sequence may be finite or infinite).

Example 3. We consider three attribute sets $A_1 = \{\Delta a\}, A_2 = \{\Delta a, \Delta b\}$ and $A_3 = \{\Delta a, \Delta b, \Delta c\}$. We obtain the families of definable sets determined by $IND(A_i)$ as the union of sets from $\Delta U/IND(A_i)$, where $i = 1, 2, 3$. In our example (see Table 1)

$$\Delta U/IND(A_1) = \{\{\Delta x_1, \Delta x_2\}, \{\Delta x_3, \Delta x_4\}, \{\Delta x_5, \Delta x_6, \Delta x_7\}, \{\Delta x_8\},$$

$$\{\Delta x_9, \Delta x_{10}, \Delta x_{11}, \Delta x_{12}\}\},$$

$$\Delta U/IND(A_2) = \{\{\Delta x_1\}, \{\Delta x_2\}, \{\Delta x_3, \Delta x_4\}, \{\Delta x_5\}, \{\Delta x_6\}, \{\Delta x_7\}, \{\Delta x_8\},$$

$$\{\Delta x_9\}, \{\Delta x_{10}\}, \{\Delta x_{11}\}, \{\Delta x_{12}\}\},$$

$$\Delta U/IND(A_3) = \{\{\Delta x_1\}, \{\Delta x_2\}, \{\Delta x_3, \Delta x_4\}, \{\Delta x_5\}, \{\Delta x_6\}, \{\Delta x_7\}, \{\Delta x_8\},$$

$$\{\Delta x_9\}, \{\Delta x_{10}\}, \{\Delta x_{11}\}, \{\Delta x_{12}\}\}.$$

Now, let us consider an approximation of changes Δf of real valued function f relative to $IND(A_i)$ [12, 13].The quality of approximation of Δf is relative to the family of definable sets determined by $IND(A_i)$ and to the acceptable deviation ϵ of Δf on relevant patterns [12, 13]. The approximation quality can be measured, e.g., by the relative size of the boundary region of approximation [12, 13].

Let $\epsilon, \delta > 0$ be given thresholds. We say that the (ϵ, δ)-derivative of f relative to the family $\{A_i\}$ exists if and only if there exists an indiscernibility relation $IND(A_j)$ in this family such that the quality of approximation of Δf [12, 13] is at least δ assuming that in the approximation were used patterns defined by cartesian products of definable sets over $IND(A_i)$ and intervals of reals with length at most ϵ.

The intuition behind this definition is the following: The derivative of a function specified by a sample (of points of its graph) exists if and only if there exists an approximation space (in a given family) allowing us to approximate the changes of function with the high quality by patterns on which the deviation of function changes is small.

Observe that the method of construction of trajectory approximation outlined before may be applied to derivatives of functions (relations) and a given elementary granule, e.g., defined by a new object. The resulting trajectory approximation may be treated as a solution of a rough differential equation determined by derivative of a transition (function) relation specified on a sample of pairs of objects.

The illustrated approach can be extended on arbitrary families of indiscernibility (similarity) relations and, more generally, on families of approximation spaces considered in [12, 13]. For example, in the case of families of indiscernibility relations one should take into account all possible nondecreasing chains of indiscernibility relations. A step toward considering rough integrals is included in [17].

Conclusions

We discussed some aspects of approximate reasoning about changes from data and domain knowledge. This paper can also be treated as a step toward developing rough calculus. The presented idea can be extended on more complex granules than elementary granules defined by indiscernibility (similarity) relations.

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