State Equation for Interval-Timed Petri Nets

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Extended Abstract

Short Introduction

Timed Petri Nets (also called Duration Petri Nets, DPNs) [7, 8] are a commonly used class for modeling formally complex technical systems as embedded real time systems, for instance in aeronautics. Engineers stated that intervals allowing some variable duration of events would be a more realistic view, so Interval-Timed Petri Nets (ITPNs) have been proposed as generalizations in [2].

Hierarchical extensions and behavioral analysis of those ITPNs under partial order aspects has been studied in [1, 4]. Timed Petri Nets, as considered in [6] are a proper subclass of ITPNs, when both classes have their duration limits as natural resp. rational, numbers. Interval-Timed Petri Nets can be simulated by or translated into other time dependent net classes, as in causal time nets [3,4] or Time Petri Nets [5]. The state space explosion makes direct reachability analysis of Interval-Timed Petri Nets non efficient, in particular to prove non-reachability, which implies to build the whole state graph. The approach to prove the non-reachability on the timeless skeleton of the ITPN only yields a sufficient but not necessary condition, not taking in account the time constraints of the net, thus this is not very satisfactory, too. Translation in causal or Time Petri Nets does not bring more efficiency. For other, simpler net classes, an algebraic way exists to calculate non-reachability, using the state equation of the net. Such state equations can be calculated for usual timeless Petri nets and DPNs, cf. [6].

In this paper, we propose a generalization to ITPNs of the time dependent state equation for DPNs from [6] which needs to be more sophisticated, taking into account each possible actual duration of a transition. Our result may look more complicated as it actually is, and is practically computable, because the used matrices are sparse matrices, i.e. their elements are mainly zeros.

Some definitions

As usual, $\mathbb{N}$ denotes the set of all natural numbers including zero and $\mathbb{Q}_0^+$ the set of all non-negative rational numbers.

A (marked) *Petri net* (PN) is a quadruple $\mathcal{N} = (P, T, v, m_0)$, if P (the set of places) and T (the set of transitions) are finite and disjoint sets, $v : (P \times T) \cup (T \times P) \longrightarrow \mathbb{N}$ defines the arcs with their weight and $m_o : P \longrightarrow \mathbb{N}$ fixes the initial marking.

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Let $\mathcal{N}$ be a PN and $D : T \longrightarrow \mathbb{Q}_0^+ \times \mathbb{Q}_0^+$ be a function. Then, a pair $\mathcal{Z} = (\mathcal{N}, D)$ is called an *Interval-Timed Petri net* (ITPN) where $\mathcal{N}$ is its *skeleton* and D its *duration function.* Thus, D defines for each transition an interval, within its firing duration can vary.

It is easy to see that without loss of generality we may consider ITPNs with $D : T \longrightarrow \mathbb{N} \times \mathbb{N}$. Therefore, only such time functions D will be considered subsequently. Thus, the interval bounds are natural numbers, but elapsing time can be considered as continuous. The bounds $sd(t)$ and $ld(t)$ with $D(t) = \big(sd(t), ld(t)\big)$ are called *shortest duration* for t and *largest duration* for t, respectively. Furthermore, each $d_i \in \big(D(t_i) \cap \mathbb{Q}_0^+\big)$ can be the actual duration of transition's t_i firing.

An ITPN behaves similarly to a PN with maximal step semantics but excluding auto-concurrency. Note that the token(s) will reach the post-set of a transition t_i only after some time elapsed, corresponding to the actual duration of this transition. The exact value of the actual duration d_i is unknown at the beginning of the firing of t_i. During the firing, the transition may stop to fire after an arbitrary time elapsing within the interval $D(t_i)$, and this value becomes d_i.

Our goal is to give an algebraic description, precisely, a linear equation, for each firing sequence. We consider in this paper only such ITPN, where for each transition t_i the duration of firing d_i is an integer. This can be supposed without loss of generality.

As usual we describe the situations in a Timed PN (DPN) and therefore also in a ITPN using states. A state s is a pair of two vectors m and u. The p-marking m is the well-known marking fixing the number of tokens in each place in the net. A t-marking u is a vector showing whether a transition is firing or not. For a firing transition the corresponding value in the t-marking shows the maximal remaining firing time of the transition.

In order to describe the relation between tokens and time, we use a generalization of the m-marking, called *time marking.* A time marking is a matrix. The number of rows is equal to the number of places and the number of columns, d equals the maximum of all largest durations in the considered ITPN, increased by 1. Each column can be considered as a p-marking. The first column represents the number of tokens in each place. These tokens are not involved in firing. The second column represents tokens which are on their way to the places and will arrive there in one time unit (one tic) later. The same is true for the third column for arriving in two time units (two tics), etc. Thus a *state* s is now defined as a pair (m, u), where m is a time marking and u is a t-marking.

A ITPN can change from one state into another one by firing of a maximal step of transitions or by time elapsing, seen as discrete time-tics. In order to establish formally the semantics, we first have to distinguish the start firing event $[\mathtt{t}$ and the stop firing event $\mathtt{t}\rangle$ for each transition t. A tic event (denoted as $\checkmark$) is enabled iff there is no firing event, neither start firing nor stop firing. Upon occurring, a tic event decrements the remaining firing time for all active firing transitions. Hence, tic events are global. During the execution of $\mathcal{Z}$ maximal steps and single tic events alternate, Note that a maximal step can be the empty step as well as a global step.

Precisely, a global step, as introduced in [6], is a (possibly empty) multiset of transitions resulting from the union of maximal steps which have to fire one after another without time elapsing (without being separated by tics). This is only possible when in each of these maximal steps there is at least one transition with actual firing duration zero. Hence, a global step is in [6] a multiset of transitions starting to fire in a certain state.

In this paper we need to generalize the notion of global step. Additionally to the multiset of all transition starting to fire at a certain state, i.e. a multiset of start firing events, here denoted by $\mathfrak{G}_{[}$, we consider also a multiset of all transitions ending its firing in this state, denoted by $\mathfrak{G}_{\rangle}$ i.e. a multiset of stop firing events. Thus, a global step is given by $\mathfrak{G} = \mathfrak{G}_{[} \cup \mathfrak{G}_{\rangle}$. A firing sequence is an alternative sequence of global steps and tics.

Let $\mathcal{Z}$ be a ITPN and

$$s^{(0)} \xrightarrow{\mathfrak{G}_1} s^{(1)} \xrightarrow{\checkmark} \tilde{s}^{(1)} \xrightarrow{\mathfrak{G}_2} s^{(2)} \xrightarrow{\checkmark} \dots \xrightarrow{\mathfrak{G}_n} s^{(n)} \xrightarrow{\checkmark} \tilde{s}^{(n)} \tag{1}$$

a firing sequence together with the states $s^{(i)} = \big(m^{(i)}, u(i)\big)$ and $\tilde{s}^{(i)} = \big(\tilde{m}^{(i)}, \tilde{u}(i)\big)$ successively reached during the firing of this sequence in $\mathcal{Z}$. Then the following equations hold for all time markings $m^{(n)}$ and $\tilde{m}^{(n)}$:

$$m^{(n)} = m^{(0)} \cdot R^{n-1} + \sum_{i=1}^{n} C^{(i)} \cdot G^{(i)} \cdot R^{n-i} \qquad \text{and} \tag{2}$$

$$\tilde{m}^{(n)} = m^{(0)} \cdot R^{n} + \sum_{i=1}^{n} C^{(i)} \cdot G^{(i)} \cdot R^{(n+1)-i} \tag{3}$$

where R is a (d, d) - matrix (the progress matrix), $C^{(i)}$ is a $(|P|, d \cdot |T|)$ - matrix (the improved time incidence matrix w.r.t. $\mathfrak{G}_i$) and $G^{(i)}$ is a $((d \cdot |T|, d))$-matrix (the bag matrix w.r.t. $\mathfrak{G}_i$).

The matrix $C^{(i)}$ is a product of the time incidence matrix C which is also a $(|P|, d \cdot |T|)$ - matrix and the matrix $U^{(i)}$ is a $(d \cdot |T|, d \cdot |T|)$ - matrix.

The time incidence matrix C is a generalization of the incidence matrix and it consists actually of $|T|$ sub-matrices of type $(|P|, d)$. All these sub-matrices describe the relation between the structure of the ITPN and the firing time durations. For each transition one proper sub-matrix is defined in the following way: When the transition t_k has the pre-place p_i then the element in the i-th row and in the 1st column (the $(i, 1)$-element) in the k-th sub-matrix is $-v(p_i, t_k)$. When the transition t_k has a post-place for p_r then the $(r, ld(t_k))$-element in the k-th sub-matrix is $v(t_k, p_r)$. Thus, the matrix C presents the relationship between the structure of the net and the maximal durations of all transitions.

The crucial point was to find an appropriated definition of matrices $U^{(i)}$. For each i, the matrix $U^{(i)}$ helps to present the actual time duration of the transition t_i, which enters in the definition of the matrix $C^{(i)}$ yielding the expected values.

Each $U^{(i)}$ is a diagonal matrix derived from $\mathfrak{G}_i$ and all elements in the diagonal are 1 or 0.

Finally, the equations (2) and (3) can also be written in the following form:

$$m^{(n)} = m^{(0)} \cdot R^{n-1} + \sum_{i=1}^{n} C \cdot U^{(i)} \cdot G^{(i)} \cdot R^{n-i} \qquad \text{and}$$

$$\tilde{m}^{(n)} = m^{(0)} \cdot R^{n} + \sum_{i=1}^{n} C \cdot U^{(i)} \cdot G^{(i)} \cdot R^{(n+1)-i} \qquad \text{respectivelly.}$$

The proof of (2) and (3) is done by induction on n.

The full paper will contain a significative running example to illustrate the construction of the proposed state equations, which also shows its feasibility. Our most original contribution in establishing the state equations is to generalize the global steps and to find an appropriate definition for the matrices $U^{(i)}$.

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