Liveness and Reachability for Elementary Object Systems

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Abstract. This contribution presents the decidability results for the formalism of Elementary Object Systems (EOS). Object nets are Petri nets which have Petri nets as tokens – an approach known as the nets-within-nets paradigm.

In this paper we study the relationship of the reachability and the liveness problem. We prove that both problems are undecidable for EOS (even for the subclass of conservative EOS) while it is well known that both are decidable for classical p/t nets. Despite these undecidability results, boundedness can be decided for conservative EOS using a monotonicity argument similar to that for p/t nets.

Keywords: Petri nets, nets-within-nets, nets as tokens, object nets, reachability, liveness, boundedness

1 Introduction

Object Systems are Petri nets which have Petri nets as tokens – an approach which is called the *nets-within-nets* paradigm, proposed by [1] for a two levelled structure and generalised in [2] nesting structures. The Petri nets that are used as tokens are called net-tokens. Net-tokens are tokens with internal structure and inner activity. This is different from place refinement, since tokens are transported while a place refinement is static. Net-tokens are some kind of *dynamic* refinement of states. The algebraic extension of objects nets – discussed in [3] – even allows operations on the net-tokens, like sequential or parallel composition. This is a concise way to express the self-modification of net-tokens at run-time in an algebraic setting.

It is quite natural to use object nets to model mobility and mobile agents (cf. [4]). Each place of the system net describes a location that hosts agents, which are net-tokens. Mobility can be modelled by moving the net-token from one place to another. This hierarchy forms a useful abstraction of the system: on a high level the agent system and on a lower level of the hierarchy the agent itself.

Approaches related to object nets are minimal object nets [5], nested nets [6], hypernets [7], AHO systems [8], adaptive workflow nets [9], and ν-Abstract Petri nets [10].

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In this paper we study decidability of the reachability and the liveness problem for Eos. There are some articles considering decidability questions of extended net-formalisms: In [5] it is shown that reachability is undecidable for Petri nets that can arbitrarily create fresh object identities. Similar results are given in [10]. Note that Eos do not have identities, so these results do not carry over to Eos. Decidability questions concerning object nets with coloured tokens, called nested nets, are studied in [6].

The paper has the following structure: Section 2 defines elementary object systems (Eos). Section 3 studies decidability problems for Eos, namely: reachability, liveness and boundedness. Section 4 studies the same problems for Conservative Eos which are restricted in a way that object nets are copied or fused but never created or destroyed. It will turn out that this restriction regains the monotonicity of the firing rule which is lost in the general case. The paper ends with an overview of the results obtained.

2 Elementary Object Systems

In the following we use standard notations for multisets and p/t nets. A multiset $\mathbf{m}$ on the set D is a mapping $\mathbf{m} : D \to \mathbb{N}$. On multisets we have the usual operations like cardinality $|\mathbf{m}|$, partial ordering: $\mathbf{m}_1 \leq \mathbf{m}_2$, addition: $\mathbf{m}_1 + \mathbf{m}_2$, etc. which are all defined pointwise. The set of all finite multisets over the set D is denoted $MS(D)$.

A *p/t net* N is a tuple $N = (P, T, \mathbf{pre}, \mathbf{post})$, such that P is a set of places, T is a set of transitions, with $P \cap T = \emptyset$, and $\mathbf{pre}, \mathbf{post} : T \to MS(P)$ are the pre- and post-condition functions. A marking of N is a multiset of places: $\mathbf{m} \in MS(P)$.

An elementary object system (Eos) is composed of a system net, which is a p/t net $\widehat{N} = (\widehat{P}, \widehat{T}, \mathbf{pre}, \mathbf{post})$ and a set of object nets $\mathcal{N} = \{N_1, \ldots, N_n\}$, which are p/t nets given as $N = (P_N, T_N, \mathbf{pre}_N, \mathbf{post}_N)$. In extension we assume that all sets of nodes (places and transitions) are pairwise disjoint. Moreover we assume $\widehat{N} \notin \mathcal{N}$. We assume the existence of the object net $\bullet \in \mathcal{N}$ which has no places and no transitions and is used to model anonymous, so called black tokens.

The system net places are typed by the mapping $d : \widehat{P} \to \mathcal{N}$ with the meaning, that the place $\widehat{p}$ of the system net contains net-tokens of the object net type N if $d(\widehat{p}) = N$. No place of the system net is mapped to the system net itself since $\widehat{N} \notin \mathcal{N}$.

Since the tokens of an Eos are instances of object nets a *marking* $\mu \in \mathcal{M}$ of an Eos OS is a *nested* multiset.

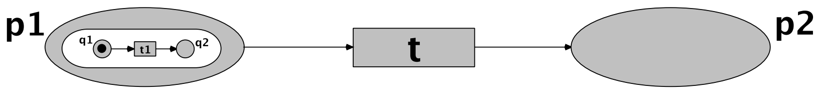
A marking of an Eos OS is denoted $\mu = \sum_{k=1}^{|\mu|} (\widehat{p}_k, M_k)$, where $\widehat{p}_k$ is a place in the system net and M_k is the marking of the net-token of type $d(\widehat{p}_k)$. To emphasise the nesting, markings are also denoted as $\mu = \sum_{k=1}^{|\mu|} \widehat{p}_k[M_k]$. Tokens of the form $\widehat{p}[\mathbf{0}]$ and $d(\widehat{p}) = \bullet$ are abbreviated as $\widehat{p}[]$.

The set of all markings which are syntactically consistent with the typing d is denoted $\mathcal{M}$ (Here $d^{-1}(N) \subseteq \widehat{P}$ is the set of system net places of the type N):

$$\mathcal{M} := MS\left(\bigcup\nolimits_{N\in\mathcal{N}} \left(d^{-1}(N) \times MS(P_N)\right)\right) \tag{1}$$

The partial order $\sqsubseteq$ on nested multisets is: $\mu_1 \sqsubseteq \mu_2$ iff $\exists \mu : \mu_2 = \mu_1 + \mu$.

Analogously to markings, which are nested multisets μ, the events of an Eos are also nested. An Eos allows three different kinds of events (cf. the following Eos).



1. System-autonomous: The system net transition t fires autonomously which moves the net-token from p_1 to p_2.
2. Object-autonomous: The object net fires transition t_1 moving the black token from q_1 to q_2. The object net itself remains at its location p_1.
3. Synchronisation: The system net transition t fires synchronously with t_1 in the object-net. Whenever synchronisation is demanded then autonomous actions are forbidden.

These three kinds of events can be described in a uniform way, namely as synchronisations: $\widehat{t}[\vartheta]$, where $\widehat{t}$ is the transition that fires in the system net and $\vartheta(N)$ is a multiset of its transitions, which have to fire synchronously with $\widehat{t}$, i.e. $\vartheta(N) \in MS(T_N)$ for each object net $N \in \mathcal{N}$.[1]

Obviously system-autonomous events are a special case of synchronous events, where $\vartheta(N) = \mathbf{0}$ for all object nets N. To describe object-autonomous events we assume the set $\{id_{\widehat{p}} \mid \widehat{p} \in \widehat{P}\}$ of *idle transitions* to be included in the set of system net transitions $\widehat{T}$, where $id_{\widehat{p}}$ formalises object-autonomous firing on the place $\widehat{p}$:

1. Each idle transitions $id_{\widehat{p}}$ has $\widehat{p}$ as its side condition: $\mathbf{pre}(id_{\widehat{p}}) = \mathbf{post}(id_{\widehat{p}}) := \widehat{p}$.
2. Each idle transition $id_{\widehat{p}}$ synchronises only with one transition from $N = d(\widehat{p})$:

$$\begin{aligned}\forall \widehat{\tau}[\vartheta] \in \Theta : \widehat{\tau} = id_{\widehat{p}} \Longrightarrow \forall N \in \mathcal{N} : |\vartheta(N)| \leq 1 \wedge \\ (\vartheta(N) \neq \mathbf{0} \iff N = d(\widehat{p}))\end{aligned} \tag{2}$$

With these idle transitions all three kinds of events are described as a synchronisation event $\widehat{\tau}[\vartheta]$, where $\widehat{\tau}$ is either a "real" transition $\widehat{t}$ or $id_{\widehat{p}}$ for some $\widehat{p}$.

[1] In the graphical representation the events are generated by transition inscriptions. For each object net $N \in \mathcal{N}$ a system net transition $\widehat{t}$ is labelled with a multiset of channels $\widehat{l}(\widehat{t})(N) = ch_1 + \cdots + ch_n$ which is depicted as $\langle N{:}ch_1, N{:}ch_2, \ldots\rangle$. Similarly, an object net transition t may be labelled with a channel $l_N(t) = ch$ – depicted as $\langle{:}ch\rangle$ whenever there is such a label. We obtain an event $\widehat{t}[\vartheta]$ by setting $\vartheta(N) := t_1 + \cdots + t_n$ to be any transition multiset such that labels match: $l_N(t_1) + \cdots + l_N(t_n) = \widehat{l}(\widehat{t})(N)$.

Definition 1 (EOS). *An* elementary object system $OS = (\widehat{N}, \mathcal{N}, d, \Theta, \mu)$ *is a tuple such that:*

1. $\widehat{N}$ *is a p/t net, called the* system net.
2. $\mathcal{N}$ *is a finite set of disjoint p/t nets, called* object nets.
3. $d : \widehat{P} \to \mathcal{N}$ *is the typing of the system net places.*
4. Θ *is the set of events.*
5. $\mu_0 \in \mathcal{M}$ *is the initial marking.*

An Eos *is* conservative *iff so is its typing d. A typing is called* conservative *iff for each place in the preset of a system net transition* $\widehat{t}$ *there is place in the postset being of equal type (except for the type* $\bullet$*):* $(d(^\bullet\widehat{t}) \cup \{\bullet\}) \subseteq (d(\widehat{t}^\bullet) \cup \{\bullet\})$.

Firing Rule Let μ be a marking of an Eos. The projection Π^1 on the first component abstracts away the substructure of all net-tokens:

$$\Pi^1 \left(\sum\nolimits_{k=1}^{|\mu|} \widehat{p}_k[M_k] \right) := \sum\nolimits_{k=1}^{|\mu|} \widehat{p}_k \tag{3}$$

The projection Π^2_N on the second component is the abstract marking of all net-tokens of the type $N \in \mathcal{N}$ ignoring their local distribution within the system net.

$$\Pi^2_N \left(\sum\nolimits_{k=1}^{|\mu|} \widehat{p}_k[M_k] \right) := \sum\nolimits_{k=1}^{|\mu|} \mathbf{1}_N(\widehat{p}_k) \cdot M_k \tag{4}$$

where the indicator function $\mathbf{1}_N : \widehat{P} \to \{0,1\}$ is $\mathbf{1}_N(\widehat{p}) = 1$ iff $d(\widehat{p}) = N$. Note that $\Pi^2_N(\mu)$ results in a marking of the object net N.

A system event $\widehat{\tau}[\vartheta]$ removes net-tokens together with their individual internal markings. Firing the event replaces a nested multiset $\lambda \in \mathcal{M}$ that is part of the current marking μ, i.e. $\lambda \sqsubseteq \mu$, by the nested multiset ρ. Therefore the successor marking is $\mu' := (\mu - \lambda) + \rho$. The enabling condition is expressed by the *enabling predicate* ϕ_{OS} (or just ϕ whenever OS is clear from the context):

$$\begin{aligned} \phi(\widehat{\tau}[\vartheta], \lambda, \rho) \iff\ & \Pi^1(\lambda) = \mathbf{pre}(\widehat{\tau}) \wedge \Pi^1(\rho) = \mathbf{post}(\widehat{\tau}) \wedge \\ & \forall N \in \mathcal{N} : \Pi^2_N(\lambda) \geq \mathbf{pre}_N(\vartheta(N)) \wedge \\ & \forall N \in \mathcal{N} : \Pi^2_N(\rho) = \Pi^2_N(\lambda) - \mathbf{pre}_N(\vartheta(N)) + \mathbf{post}_N(\vartheta(N)) \end{aligned} \tag{5}$$

With $\widehat{M} = \Pi^1(\lambda)$ and $\widehat{M'} = \Pi^1(\rho)$ as well as $M_N = \Pi^2_N(\lambda)$ and $M'_N = \Pi^2_N(\rho)$ for all $N \in \mathcal{N}$ the predicate ϕ has the following meaning:

1. The first conjunct expresses that the system net multiset $\widehat{M}$ corresponds to the pre-condition of the system net transition $\widehat{t}$, i.e. $\widehat{M} = \mathbf{pre}(\widehat{t})$.
2. In turn, a multiset $\widehat{M'}$ is produced, that corresponds with the post-set of $\widehat{t}$.
3. An object net transition τ_N is enabled if the combination M_N of the markings net-tokens of type N enable it, i.e. $M_N \geq \mathbf{pre}_N(\vartheta(N))$.

4. The firing of $\widehat{\tau}[\vartheta]$ must also obey the *object marking distribution condition* which is essential for the formulation of linear invariants: $M'_N = M_N - \mathbf{pre}_N(\vartheta(N)) + \mathbf{post}_N(\vartheta(N))$, where $\mathbf{post}_N(\vartheta(N)) - \mathbf{pre}_N(\vartheta(N))$ is the effect of the object net's transition on the net-tokens.

Note that (1) and (2) assures that only net-tokens relevant for the firing are included in λ and ρ. Conditions (3) and (4) allows for additional tokens in the net-tokens.

For system-autonomous events $\widehat{t}[\vartheta_{id}]$ the enabling predicate ϕ can be simplified further. We have $\mathbf{pre}_N(id_N) = \mathbf{post}_N(id_N) = \mathbf{0}$. This ensures $\Pi^2_N(\lambda) = \Pi^2_N(\rho)$, i.e. the sum of markings in the copies of a net-token is preserved w.r.t. each type N. This condition ensures the existence of linear invariance properties (cf. [11]).

Analogously, for an object-autonomous event we have an idle-transition $\widehat{\tau} = id_{\widehat{p}}$ for the system net and the first and the second conjunct is: $\Pi^1(\lambda) = \mathbf{pre}(\widehat{t}) = \widehat{p} = \mathbf{post}(\widehat{t}) = \Pi^1(\rho)$. So, there is an addend $\lambda = \widehat{p}[M]$ in μ with $d(\widehat{p}) = N$ and M enables $t_N := \vartheta(N)$.

Definition 2 (Firing Rule). *Let OS be an* EOS *and $\mu, \mu' \in \mathcal{M}$ markings. The event $\widehat{\tau}[\vartheta]$ is enabled in μ for the mode $(\lambda, \rho) \in \mathcal{M}^2$ iff $\lambda \sqsubseteq \mu \wedge \phi(\widehat{\tau}[\vartheta], \lambda, \rho)$ holds.*

An event $\widehat{\tau}[\vartheta]$ that is enabled in μ for the mode (λ, ρ) can fire: $\mu \xrightarrow[OS]{\widehat{\tau}[\vartheta](\lambda,\rho)} \mu'$. The resulting successor marking is defined as $\mu' = \mu - \lambda + \rho$.

We write $\mu \xrightarrow[OS]{\widehat{\tau}[\vartheta]} \mu'$ whenever $\mu \xrightarrow[OS]{\widehat{\tau}[\vartheta](\lambda,\rho)} \mu'$ for some mode (λ, ρ).

Note that the firing rule has no a-priori decision how to distribute the marking on the generated net-tokens. Therefore we need the mode (λ, ρ) to formulate the firing of $\widehat{\tau}[\vartheta]$ in a functional way.

As for p/t nets the firing rule has nice properties (cf. [11] for details):

- Firing is reversible, i.e. for each EOS OS we obtain OS^{rev} by inverting the arcs and have $\mu_1 \xrightarrow[OS]{\theta} \mu_2 \iff \mu_2 \xrightarrow[OS^{rev}]{\theta^{rev}} \mu_1$.
- The behaviour of the system net in the EOS when ignoring the net-tokens structure cannot be distinguished from the system net $\widehat{N}$ as a p/t net, i.e. $\mu \xrightarrow[OS]{\widehat{t}[\vartheta]} \mu' \Longrightarrow \Pi^1(\mu) \xrightarrow[\widehat{N}]{\widehat{t}} \Pi^1(\mu')$.
- The invariance calculus for p/t nets can be extended to EOS in a compositional way, i.e. invariance equations can be obtained from the invariance equations of the constituting components separately.

3 Decidability Problems for EOS

The interesting part in the firing rule of EOS is the fact that moving an object net-token in the system net has the power to modify the state of an unbounded number of tokens, i.e. all the tokens of the object net-tokens (including the case of zero tokens). It is therefore a natural question whether this increases the expressiveness of Eos compared to p/t nets. Here we consider the most well known decidability problems for Petri nets: The reachability, the liveness and the boundedness problem.

In the following we give a direct Eos-simulation of inhibitor nets.

Proposition 1. *For each inhibitor net N^* there is an simulating* Eos $OS_{strong}(N^*)$.

Proof. We show that each inhibitor net can be simulated by the Eos $OS_{strong}(N^*)$. Without loss of generality we consider inhibitor nets without arc weights and we assume that for each transition we have that whenever a place p is connected via a inhibitor arc then this place is not connected with t via a normal arc. Let us consider a inhibitor net given as $N^* = (P^*, T^*, F^*, F^*_{inh}, m_0)$, where $F^*_{inh} \subseteq P^* \times T^*$ describes the inhibitor arcs. A transition t is enabled in m iff there is at least one token on each input place and all inhibitor place carry the empty marking, i.e. $m(p) \geq F(p,t)$ for all p and $m(p) = 0$ for all p such that $(p,t) \in F^*_{inh}$.

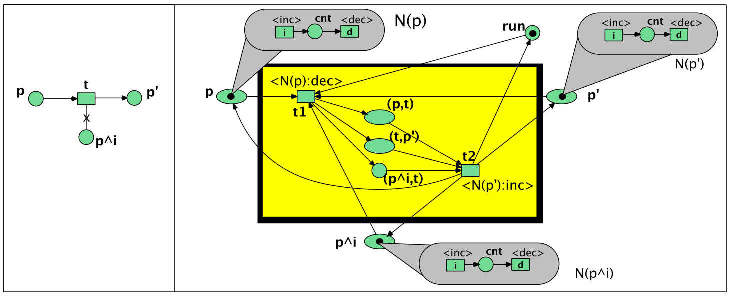


Fig. 1. The Eos-translation of inhibitor nets

Each marking m of the inhibitor net is encoded as the marking $\mu(m)$ of the Eos. We say that a nested marking μ encodes a marking m of N^* whenever μ contains exactly one net-token on each place $p \in P^*$ (and none on the other places) and the net-token on p has exactly $m(p)$ tokens on its place cnt:

$$\mu(m) := \mathsf{run}[] + \sum_{p \in P^*} p[m(p) \cdot \mathsf{cnt}_{d(p)}]$$

Each firing $m \xrightarrow{t} m'$ is simulated deterministically by the firing $\mu(m) \xrightarrow{t_1 t_2} \mu(m')$.

The simulating Eos $OS_{strong}(N^*) = (\widehat{N}, \mathcal{N}, d, \Theta, \mu_0)$ is constructed in the following way:

- For each place $p \in P^*$ in the inhibitor net the simulating Eos has one object-net $N(p)$. Each object-net $N(p)$ has exactly one place $\mathsf{cnt}_{N(p)}$ and the two transitions $\mathsf{i}_{N(p)}$ and $\mathsf{d}_{N(p)}$, where $\mathsf{i}_{N(p)}$ is labelled with channel $\mathsf{inc}_{N(p)}$ and

$\mathsf{d}_{N(p)}$ is labelled with channel $\mathsf{dec}_{N(p)}$. In particular, all the object nets $N(p)$ have the same net structure. Additionally we have the object-net $\bullet$:

$$\mathcal{N} = \{\bullet\} \cup \{N(p) \mid p \in P^*\}$$

- The system net $\widehat{N}$ is obtained from the inhibitor N^* via a substitution for each transition which is illustrated in Figure 1:
  Each transition $t \in T^*$ is replaced by the two transitions t_1 and t_2.

  $$\widehat{T} := \{t_1, t_2 \mid t \in T^*\}$$

  For each input arc $(p,t) \in F^* \cap (P^* \times T^*)$ we add the place (p,t); for each output arc $(t,p') \in F^* \cap (T^* \times P^*)$ we add the place (t,p'); for each inhibitor arc $(p^i,t) \in F^*_{inh}$ we add the place (p^i,t). Additionally, we have one global *run* place which guarantees that firing of t_1 must be followed by t_2 before any other transition can fire.

  $$\widehat{P} := P^* \cup F^* \cup F^*_{inh} \cup \{\mathsf{run}\}$$

  For each input arc (p,t) the transition t_1 is labelled with $\mathsf{dec}_{N(p)}$:

  $$\widehat{l}(t_1)(N(p)) = \begin{cases} \mathsf{dec}_{N(p)}, & \text{if } (p,t) \in F^* \cap (P^* \times T^*) \\ \mathbf{0}, & \text{otherwise} \end{cases}$$

  Analogously, for each output arc (t,p') the transition t_2 is labelled with $\mathsf{inc}_{N(p')}$.
- The typing d is defined as:

  $$d(p) = N(p) \qquad d(p,t) = N(p) \qquad d(t,p') = N(p') \qquad d(p^i,t) = d(\mathsf{run}) = \bullet$$

- The initial marking is defined as the encoding of m, i.e. $\mu_0 := \mu(m_0)$.

Whenever a place p^i is connected via a inhibitor arc with t then t_1 has exactly one place of type $N(p^i)$ in its preset but none in the postset. Therefore t_1 can only fire if the marking of the net-token is the empty multiset. Whenever t_2 fires it generates one net-token on p^i again which must be empty since there is no place of type $N(p^i)$ in the preset of t_2. It is straightforward to see that we have:

$$m \xrightarrow{t} m' \iff \mu(m) \xrightarrow{t_1 t_2} \mu(m')$$

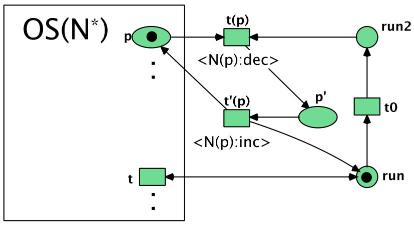
This proves that the Eos $OS_{strong}(N^*)$ simulates the inhibitor net N^*. □

We define the liveness problem for Eos analogously to that of p/t nets: For the liveness problem one has to decide whether all events $\theta \in \Theta$ of a given Eos *OS* are live. An event θ is live if for all markings μ reachable from μ_0 there exists a marking μ' reachable from μ that enables θ.

Proposition 2. *Reachability is reducible to liveness for* Eos.

Proof. The proof follows the idea given in [12] that shows the equivalence of reachability and liveness for p/t nets.

It is sufficient to consider the problem whether the empty marking is reachable since for each inhibitor net N_1 and each marking m we can construct another inhibitor net N_2 with the property: The marking m is reachable in N_1 iff $\mathbf{0}$ is reachable in N_2. The net N_2 is obtained from N_1 by adding one place run and one transition t. The additional run-place is attached as a side condition to each transition of N_1. Initially the place run is marked with one token. The additional transition t removes exactly $m(p)$ tokens from each p (where m is the given marking tested for reachability) and one token from run. The postset of t is empty. It is obvious that N_2 has the desired property.



We will construct an Eos $OS(N^*)$ from a given p/t net N^* such that the empty marking is reachable in N^* iff the event $t_0[\vartheta]$ is not live in $OS(N^*)$.

Assume the inhibitor net is given as $N^* = (P^*, T^*, F^*, F^*_{inh}, m_0)$. We define $OS(N^*)$ almost the same as in Prop. 1. We add transition t_0 and the place run_2 and for each $p \in P^*$ the place p' and the transitions $t(p)$ and $t'(p)$. We set $d(\mathsf{run}_2) := \bullet$ and $d(p') := d(p) = N(p)$. Remark: Since t_0 has only places of the black token type in the pre- and postset (i.e. $\mathcal{N}(t_0) = \{\bullet\}$) we obtain that if the event $t_0[\vartheta]$ is activated then ϑ is uniquely determined as $\vartheta(N) = \mathbf{0}$ holds for all $N \in \mathcal{N}(t_0)$.

As before, we define $\mu(m)$ as: $\mu(m) := \mathsf{run}[] + \sum_{p \in P^*} p[m(p) \cdot \mathsf{cnt}_{d(p)}]$

A marking that is reachable in N^* is so in $OS(N^*)$: Assume that $\mathbf{0}$ is reachable in N^*. In $\mu(\mathbf{0})$ we have $\mu(\mathbf{0}) \xrightarrow{t_0[\vartheta]} \mu := \mathsf{run}_2[] + \sum_{p \in P^*} p[\mathbf{0}]$ and in μ no event is activated anymore. So, if $\mathbf{0}$ is reachable in N^* then clearly $t_0[\vartheta]$ is not live.

Assume that $\mathbf{0}$ is not reachable in N^*. Then $t_0[\vartheta]$ is live: For each marking $m^* \neq \mathbf{0}$ we have $m^*(p_0) > 0$ for some p_0 and therefore we have $\mu(m^*) \xrightarrow{t_0[\vartheta]} \mu' \xrightarrow{t(p_0)[\vartheta']t'(p_0)[\vartheta'']} \mu(m^*)$. Note that $t(p_0)[\vartheta']t'(p_0)[\vartheta'']$ does not alter the marking of the net-token on p. □

It is well known that reachability, liveness and boundedness, are undecidable for inhibitor nets. Therefore, we obtain the following result.

Corollary 1. *Reachability, liveness and boundedness, are undecidable for* Eos.

4 Decidability Results for Conservative EOS

The expressiveness of Eos was due to a non-conservative typing. Recall, that a typing d is called conservative if for all t we have that for each place $\widehat{p}$ in its preset there is a place $\widehat{p}'$ in its postset typed with the same net, i.e. $d(\widehat{p}) = d(\widehat{p}')$.

It will turn out that the blocking behaviour is somehow the only source of the equivalence of Eos to inhibitor nets. If we consider conservative Eos, we will regain the monotonicity property of the firing rule, which is essential for the construction of coverability graphs.

Theorem 1. *Boundedness and Coverability are decidable for conservative* Eos.

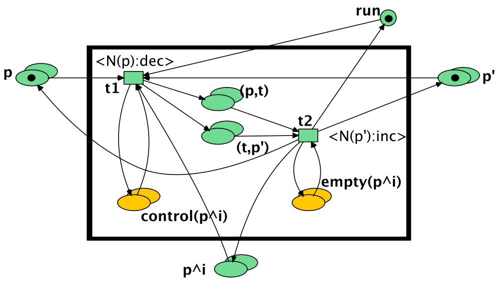
Proof. In [11] we have shown that the reachability graph of a conservative Eos is a well structured transitions system [13]. Generalising the result of Karp and Miller it is shown in [13] that the boundedness and the coverability problem are decidable for well structured transitions system. □

In the following we show that reachability and liveness remain undecidable even if we restrict Eos to conservative typings.[2] We can reuse the translation of an inhibitor net in Prop. 1. The simulation provided for conservative Eos is a very weak one, in the sense that the simulation might make wrong guesses about the test on zero, but all misguesses are stored in the marking till the end. Nevertheless for each firing sequence $m \xrightarrow{*} m'$ in the inhibitor net there is one corresponding sequence $\tilde{\mu}(m) \xrightarrow{*} \tilde{\mu}(m')$ in the simulating Eos. Additionally we have the property that for each firing $\tilde{\mu}(m) \xrightarrow{*} \mu$ such that μ does not correspond to any marking in the inhibitor net then no marking reachable from μ ever will do so.

Proposition 3. *For each inhibitor net N^* there is a conservative* Eos *$OS(N^*)$ that has the following property:*

$$m \xrightarrow[N^*]{*} m' \iff \tilde{\mu}(m) \xrightarrow[OS(N^*)]{*} \tilde{\mu}(m')$$

Proof. Let us consider a inhibitor net given as $N^* = (P^*, T^*, F^*, F^*_{inh}, m_0)$, where $F^*_{inh} \subseteq P^* \times T^*$ describes the inhibitor arcs. The simulating Eos $OS(N^*)$ is obtained by minor modifications from the Eos $OS_{strong}(N^*)$ from Prop. 1



[2] For reachability this result has already been shown in [14], where we have given a weak simulation of counter programs. But the simulation of inhibitor nets given here results in a representation that is more accessible to the liveness problem.

We say that a nested marking $\tilde{\mu}$ encodes a marking m of N^* whenever $\tilde{\mu}$ contains exactly one empty net-token on each $\mathsf{control}$ place and each empty place: $\tilde{\mu}(m) := \mu(m) + \mathsf{control}(p)[\mathbf{0}] + \mathsf{empty}(p)[\mathbf{0}]$. The initial marking is $\tilde{\mu}_0 := \tilde{\mu}(m_0)$. By construction, the simulating Eos $OS(N^*)$ is conservative.

So, we have that all the net-tokens on control-places have the empty marking if and only if all guesses on the emptiness of inhibitor places have been right during the simulation: When all guesses have been right during the simulation then the resulting marking perfectly reflects the marking m. But after the first wrong guess we never reach a marking $\tilde{\mu}$ such that it is a configuration marking $\tilde{\mu}(m)$ for some m since we can never get rid of the tokens in the net-token on the places $\mathsf{control}(p)$. □

The reduction from the reachability problem to the liveness problem is also possible for conservative Eos.

Proposition 4. *Reachability is reducible to liveness for conservative* Eos.

Proof. The construction in Prop. 2, which reduces reachability to liveness, can be adjusted to the conservative case: To each control place $\mathsf{control}(p)$ we add a side transition t_1 which is dead iff all all guesses habe been made right. Therefore, the empty marking $\mathbf{0}$ is reachable in the inhibitor net N^* iff for all $p \in P^*$ we have that $t_1[\vartheta_1]$ is not live for all ϑ_1 and $t_0[\vartheta_0]$ is live for some ϑ_0. □

Therefore, liveness is undecidable even for conservative Eos.

Corollary 2. *Reachability and liveness are undecidable for conservative* Eos.

For conservative Eos one cannot destroy or generate net-tokens. Only joining or splitting is allowed. If we restrict conservative Eos even further and do not allow a fusion (or splitting) of two or more net-tokens then we obtain the class of *Generalised State Machines* [15]. An Eos OS is a *generalised state machine* (GSM) iff for all $\widehat{t}$ there is either exactly one place in the preset and one in the postset typed with the object net N (or there are no such places) and the initial marking μ_0 has at most one net-token of each type. It is shown in [15] that for each Eos OS there exists a p/t net Rn(OS), called the *reference net*, which bi-simulates the behaviour of OS.

From a modelling point of view these results are interesting since in many scenarios net-tokens model physical entities which are neither cloned, combined, created nor destroyed. These models therefore have the GSM property. From a more theoretical point of view the correspondence of each GSM OS with its reference net Rn(OS) allows to simplify notations considerably – at the price of limiting the expressiveness. For these reasons some formalism, like the ones in [1], [7], or [9], are initially restricted to generalised state machines.

We also studied the impact of bounds on markings. In [16] we have defined four different kinds of bounds. Here we recall two notions: A marking μ is called safe(2) (or: system-safe) iff for all reachable markings there is at most one token on each system net place. A marking μ is called safe(3) (or simply: safe) iff for all reachable markings there is at most one token on each system net place and each net-token is safe:

We have shown in [16] that safe(3) Eos have finite state spaces. We know that problems, like reachability or liveness are therefore decidable for safe(3) Eos, but they are at least as complex as the corresponding problem for p/t nets. It is a known fact that most interesting questions about the behaviour of classical 1-safe p/t nets like liveness, deadlock-freedom, and reachability are Pspace-hard (see [17]). It turns out that polynomial space is also sufficient to decide reachability and liveness (cf. [16] for details) so both are PSpace-complete problems.

In general, safe(2) Eos have infinite state spaces, since the net-tokens' markings are unbounded. If we look at the construction in Prop. 2 and in Prop. 3 we can observe that the constructed Eos are already safe(2).

Corollary 3. *Reachability and liveness are undecidable for conservative, safe(2) Eos.*

5 Conclusion

This paper studies the Petri net formalism of elementary object net systems (Eos). Object nets are Petri nets which have Petri nets as tokens. Eos are called elementary since the nesting is restricted to two levels only. Interestingly enough, even for the restricted class of elementary object nets reachability, liveness, and boundedness are undecidable problems. The following table summarises the most relevant decidability results of this paper. (Here *u* denotes undecidability and *d* decidability of the problem.) Even for the class of conservative Eos – where boundedness remains decidable – the reachability and the liveness problem remain undecidable.

		conservative		system-safe and	safe
	Eos	Eos	GSM	conservative Eos	Eos
reachability	u	u	d	u	PSpace-complete
liveness	u	u	d	u	PSpace-complete
boundedness	u	d	d	d	always bounded

Additionally, we studied Eos that are –in some sense– safe systems. Only the class of safe(3) Eos has finite state spaces. The class of of safe(3) Eos is not really simpler than the general case as reachability and the liveness are still undecidable for them. On the other hand the LTL/CTL model checking problem for safe(3) Eos is as complex as the corresponding problem for p/t nets which implies that reachability and liveness are PSpace-hard problems.

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