Language Theoretic Properties of Client/Server Systems

Ludwik Czaja[1,2] and Manfred Kudlek[3]

[1] Instytut Informatyki, Uniwersytet Warszawski, PL
[2] Uczelnia Vistula (Wyższa Szkoła Ekonomiczno-Informatyczna w Warszawie), PL
email: lczaja@mimuw.edu.pl
[3] Fachbereich Informatik, MIN-Fakulät, Universität Hamburg, DE
email: kudlek@informatik.uni-hamburg.de

In [1] decision problems of deadlock and fairness for client/server systems have been considered. The model introduced there, consisted of a finite number n of processors and a finite number m of actions which each processor can handle. A processor p_i can issue an event r_{ij} - a request to execute a_j, an event s_{ij} - a start of executing a_j, and an event t_{ij} - a termination of executing a_j. If $\sqsubseteq$ denotes the temporal order represented by precedence in a sequence of events, then $r_{ij} \sqsubseteq s_{ij} \sqsubseteq t_{ij}$ where these events concern the same instance in which processor p_i applies for action a_j.

Now the model also should be concurrent or parallel in some sense.

The main results there are two theorems on conditions on subsets of the set of all event sequences to decide the deadlock and fairness problem of such parallel/concurrent systems. In particular, these theorems are related to the emptiness or finiteness problem of certain sets of events.

Let $E = \{r_{ij}, s_{ij}, t_{ij} \mid 1 \le i \le n, 1 \le j \le m\}$ denote the set of all events, and $\mathcal{L}$ a class of parallel systems. Then the following two theorems have been shown in [1] where $\odot$ denotes catenation and $A/_{\odot}\{v\} = \{u \mid vu \in A\}$:

Proposition 1: The deadlock problem for a class $\mathcal{L}$ of parallel systems is decidable iff the emptiness problem for $(L/_{\odot}\{v\}) \cap (E^* \odot \{s\} \odot E^*)$ is decidable for all $L \in \mathcal{L}$, $v \in L$, $s \in \{s_{ij} \mid 1 \le i \le n, 1 \le j \le m\}$.

Proposition 2: The fairness problem for a class $\mathcal{L}$ of parallel systems is decidable iff the emptiness problem for $(L/_{\odot}\{v\}) \setminus (E^* \odot \{s\} \odot E^*)$ is decidable for all $L \in \mathcal{L}$, $v \in L$, $s \in \{s_{ij} \mid 1 \le i \le n, 1 \le j \le m\}$.

Here we investigate the basic language theoretic properties of various concurrent/parallel systems, such as to which word or multiset language class they belong, and which decidability properties hold. As such it is a first approach to study such properties of language classes defined by several operators as $\odot$ and $\sqcup\!\sqcup$, and closure operators as $/_{\odot}$, $\cap$, and $\setminus$, as well as of corresponding classes of multiset languages.

First we consider a simple model of k actions, start a_i and termination b_i of an action i. Again, $a_i \sqsubseteq b_i$ should hold. It can be seen as the wellformedness of parentheses.

CONCURRENCY, SPECIFICATION AND PROGRAMMING
M. Szczuka et al. (eds.): Proceedings of the international workshop CS&P 2011
September 28-30, Pułtusk, Poland, pp. 79-84

The simplest case is $\{a_ib_i \mid 1 \leq i \leq k\}^* \in \mathbf{REG}$. Here one has $a_i \sqsubseteq b_i \sqsubseteq a_j$ for $1 \leq i, j \leq k$, which means that a new action can only start if a previous one terminated.

Another case is $\{a_i, b_i \mid 1 \leq i \leq k\}^{\sqcup\!\sqcup} \in \mathbf{CF}$ where $\sqcup\!\sqcup$ denotes the shuffle operator. Here one only has $a_i \sqsubseteq b_i$, which means that another action a_j, which may be the same $(j = i)$, can start before termination b_i.

A more restricted version is the condition that a new action can start only before termination of b_i if $j \neq i$. In this case one gets a set from $\mathbf{RAT}_{\sqcup\!\sqcup}$, the algebraic closure of finite languages under $(\cup, \sqcup\!\sqcup, {}^{\sqcup\!\sqcup})$.

In the sequel we consider the different concurrent/parallel systems in more detail.

For that let $\{a_i, b_i \mid 1 \leq i \leq k\}$ be an alphabet of parentheses a_i, b_i.

1: $A = \{a_ib_i \mid 1 \leq i \leq k\}^* \in \mathbf{REG}$. Then the iteration lemma for regular languages holds, and membership as well as emptiness and finiteness problems are decidable. Furthermore, $A/_{\odot}\{u\} \in \mathbf{REG}$ for all $u \in \{a_i, b_i \mid 1 \leq i \leq k\}$ where $A/_{\odot}\{u\} = \{v \mid uv \in A\}$.

2: The Dyck language $D_k = D(1, \cdots, k)$ is defined by the grammar $S \to SS$, $S \to a_iSb_i$ $(1 \leq i \leq k)$, $S \to \lambda$. Thus $D_k \in \mathbf{CF}$. Note that $D_k \notin \mathbf{REG}$ since $D_k \cap (\{a_i\}^* \odot \{b_i\}^*) = \{a_i^m b_i^m \mid m \geq 0\} \notin \mathbf{REG}$ where $\odot$ is the catenation operator.

For $k = 1$ follows $D(i) = \{a_ib_i\}^{\sqcup\!\sqcup}$ and $D(i) \sqcup\!\sqcup D(i) = D(i)$ where $\sqcup\!\sqcup$ is the shuffle operator.

Let

$$L = \coprod_{i=1}^{k} \{a_ib_i\}^{\sqcup\!\sqcup} \in \mathbf{RAT}_{\sqcup\!\sqcup} .$$

$\coprod$ denotes the finite application of $\sqcup\!\sqcup$. Thus an iteration lemma ($\sqcup\!\sqcup$) as for regular languages holds, and membership as well as emptiness problems are decidable.

Obviously $D_k \subseteq L$, $L \sqcup\!\sqcup D(i) = L$, and $L \sqcup\!\sqcup L = L$.

$L \notin \mathbf{CF}$ since

$L \cap (\{a_1\}^* \odot \{a_2\}^* \odot \{b_1\}^* \odot \{b_2\}^*) = \{a_1^m a_2^n b_1^m b_2^n\} \mid m, n \geq 0\} \notin \mathbf{CF}$ for $k > 1$ since $\#_{a_j}(w) = \#_{b_j}(w)$ for $1 \leq j \leq k$ and $w \in L$. Here $\#_a(w)$ denotes the number of a's in w.

Since $L \in \mathbf{RAT}_{\sqcup\!\sqcup}$, there is an iteration lemma and emptiness as well as finiteness problems are decidable.

3: Another characterization of L is the following one. Consider Y_i (as in [1]) defined by

$$Y_i = (\{a_i\} \odot (\bigcup_{j=1}^{k} Y_j) \odot \{b_i\})^* \ .$$

$Y_i \in \mathbf{CF}$ is generated by the grammar
$S_i \to \lambda$, $S_i \to S_i S_i$, $S_i \to a_i S_j b_i$ $(1 \le j \le k)$.

Since $S_i \to a_i S_i b_i$, $D(i) = \{a_i b_i\}^{\sqcup\!\sqcup} \subseteq Y_i$, and obviously $Y_i \subseteq D_k$.
Trivially also $\{a_i^m b_i^m \mid m \ge 0\} \subseteq Y_i$.
Since $Y_i \subseteq D_k \subseteq L$ it follows that

$$\coprod_{i=1}^{k} Y_i \subseteq \coprod_{i=1}^{k} D_k \subseteq \coprod_{i=1}^{k} L = L \ .$$

On the other hand, since $D(i) = \{a_i b_i\}^{\sqcup\!\sqcup} \subseteq Y_i$ it follows that

$$L \subseteq \coprod_{i=1}^{k} Y_i \ ,$$

hence

$$\coprod_{i=1}^{k} Y_i = \coprod_{i=1}^{k} D_k = L \ .$$

4: A slightly different system is the following one. Consider Z_i defined by

$$Z_i = (\{a_i\} \odot (\bigcup_{j \ne i} Z_j) \odot \{b_i\})^* \ .$$

$Z_i \in \mathbf{CF}$ is generated by the grammar
$S_i \to S_i S_i$, $S_i \to a_i S_j b_i$ $(j \ne i)$, $S_i \to \lambda$.
Let

$$B = \coprod_{j=1}^{k} Z_j \ .$$

$B \notin \mathbf{CF}$.
Now, for $i \ne j$, $S_i \to a_i S_j b_i \to a_i a_j S_i b_j b_i$ and $S_j \to a_j S_i b_j \to a_j a_i S_j b_i b_j$. Therefore $\{(a_i a_j)^m (b_j b_i)^m \mid m \ge 0\} \subseteq Z_i$ and $\{(a_j a_i)^n (b_i b_j)^n \mid n \ge 0\} \subseteq Z_j$.
From this follows
$B \cap (\{a_i a_j\}^* \odot \{a_j a_i\}^* \odot \{b_i b_j\}^* \odot \{b_j b_i\}^*)$
$= \{(a_i a_j)^m (a_j a_i)^n (b_i b_j)^m (b_j b_i)^n \mid m, n \ge 0\} \notin \mathbf{CF}$.

5: Both, Y_i and Z_i are least fixed point solutions, starting with $Y_i^{(0)} = \emptyset$ and $Z_i^{(0)} = \emptyset$. By induction follows $Y_i^{(m)} \subseteq Y_i^{(m+1)} \subseteq \Sigma_k^*$ and $Z_i^{(m)} \subseteq Z_i^{(m+1)} \subseteq \Sigma_k^*$ for $1 \le i \le k, m \ge 0$.

Also by induction follows $Z_i \subseteq Y_i$ for $1 \leq i \leq k$ since $Z_i^{(0)} = Y_i^{(0)} = \emptyset$, and the induction hypothesis $Z_i^{(m)} \subseteq Y_i^{(m)}$ implies

$$Z_i^{(m+1)} = (\{a_i\} \odot (\bigcup_{j \neq i} Z^{(m)}) \odot \{b_i\})^* \subseteq (\{a_i\} \odot (\bigcup_{j \neq i} Y^{(m)}) \odot \{b_i\})^*$$

$$\subseteq (\{a_i\} \odot (\bigcup_{i=1}^{k} Y^{(m)}) \odot \{b_i\})^* = Y_i^{(m+1)} \ .$$

Thus $Z_i \subseteq Y_i$.

$Z_i \subset Y_i$. This is shown by considering

$$Z_i \cap \{a_i, b_i\}^* = \{(a_i b_i)^n \mid n \geq 0\}$$

and

$$Y_i \cap \{a_i, b_i\}^* = D(i) \ .$$

Furthermore, $Y_j \cap \{a_i, b_i\}^* = \{\lambda\}$ and $Z_j \cap \{a_i, b_i\}^* = \{\lambda\}$ for $i \neq j$.

Then

$$(\coprod_{j=1}^{k} Z_j) \cap \{a_i, b_i\}^* = \{(a_i b_i)^n \mid n \geq 0\}$$

and

$$(\coprod_{j=1}^{k} Y_j) \cap \{a_i, b_i\}^* = D(i)$$

imply

$$B = \coprod_{j=1}^{k} Z_j \subset \coprod_{j=1}^{k} Y_j = L \ .$$

Note also that $\#_{a_i}(u) = \#_{b_i}(u)$ for $u \in L$, $u \in Y_j$, $u \in Z_j$, $u \in B$ and $1 \leq i, j \leq k$.

6. Since $Y_i \in \mathbf{CF}$ and $Z_i \in \mathbf{CF}$ it follows that $\Psi(Y_i)$ and $\Psi(Z_i)$ are semilinear sets, and therefore also

$$\Psi(A) = \Psi(\coprod_{j=1}^{k} Y_j) = \bigoplus_{j=1}^{k} \Psi(Y_j) \in \mathbf{RAT}_{\oplus}$$

and

$$\Psi(B) = \Psi(\coprod_{j=1}^{k} Z_j) = \bigoplus_{j=1}^{k} \Psi(Z_j) \in \mathbf{RAT}_{\oplus} \ .$$

Here Ψ denotes the Parikh vector and $\bigoplus$ the finite application of multiset addition $\oplus$, and $\mathbf{RAT}_{\oplus}$ the class of semilinear sets which also can be characterized by **mCF**, the class of multiset languages generated by context-free (regular) multiset grammars. Thus the problems of membership, emptiness and finiteness are decidable.

Thus $\Psi(L)$ and $\Psi(B)$ can be generated by context-free multiset grammars. Furthermore, for any $\mu \in \Psi(L)$ or $\mu \in \Psi(B)$, the membership problem for $\Psi(L)/_{\oplus}\{\mu\}$ and $\Psi(B)/_{\oplus}\{\mu\}$ is decidable. This follows from just adding a production $\mu \to \mathbf{0}$ to the multiset grammars. The operator $/_{\oplus}$ is defined in a similar way as $/_{\odot}$.

7: Consider
$S_i \to \lambda, S_i \to a_i b_i S_i$ generating $X_i = \{a_i b_i\}^* \in \mathbf{REG}$.

$$C = \coprod_{j=1}^{k} X_j \in \mathbf{RAT}_{\sqcup\!\sqcup} .$$

Again, $\#_{a_i}(u) = \#_{b_i}(u)$ for $u \in X_j$, $u \in C$ and $1 \leq i, j \leq k$.

Now we consider the more detailed client/server systems from [1].

8: Example (Figure 1.3 from [1]).
$R_i \to r_{ij} S_{ij}$, $S_{ij} \to s_{ij} T_{ij}$, $T_{ij} \to t_{ij} R_i$ $(1 \leq j \leq m)$
$X_{ij} = \{r_{ij} s_{ij} t_{ij}\}^* \in \mathbf{REG}$.

$$X_i = \coprod_{j=1}^{m} X_{ij} \in \mathbf{RAT}_{\sqcup\!\sqcup}$$

$$X = \coprod_{i=1}^{n} X_i \in \mathbf{RAT}_{\sqcup\!\sqcup} .$$

Again, $\#_{r_{ij}}(u) = \#_{s_{ij}}(u) = \#_{t_{ij}}(u)$ for $u \in X_{ij}$, $u \in X_i$, $u \in X$.

In the next system request and start of an action are indivisible.

9: Consider $(1 \leq i \leq n, 1 \leq j \leq m)$

$$X_{ij} = (\{r_{ij} s_{ij}\} \cdot (\bigcup_{\ell=1}^{m} X_{i\ell}) \cdot \{t_{ij}\})^* .$$

$X_{ij} \in \mathbf{CF}$ by $S_{ij} \to \lambda, S_{ij} \to S_{ij} S_{ij}, S \to r_{ij} s_{ij} S_{i\ell} t_{ij}$ $(1 \leq \ell \leq m)$.
Let

$$X_i = \coprod_{\ell=1}^{m} X_{i\ell}$$

and

$$X = \coprod_{i=1}^{n} X_i \ .$$

Also, $\#_{r_{ij}}(u) = \#_{s_{ij}}(u) = \#_{t_{ij}}(u)$ for $u \in X_{ij}$, $u \in X_i$, $u \in X$.
Then

$$\Psi(X_i) = \Psi(\coprod_{\ell=1}^{m} X_{i\ell}) = \bigoplus_{\ell=1}^{m} \Psi(X_{i\ell}) \in \mathbf{RAT}_{\oplus}$$

and

$$\Psi(X) = \Psi(\coprod_{i=1}^{n} X_i) = \bigoplus_{i=1}^{n} \Psi(X_i) \in \mathbf{RAT}_{\oplus}$$

are semilinear sets, and therefore the membership problems for $\Psi(X_i)/_{\oplus}\{\mu\}$ and $\Psi(X)/_{\oplus}\{\mu\}$ are decidable, again adding a production $\mu \to \mathbf{0}$ to the context-free multiset grammars for $\Psi(X_i)$ and $\Psi(X)$.

It remains to investigate to which language classes $A/_{\odot}$ belongs for various systems.

References

1. Czaja, Ludwik: *On Deadlock and Fairness Decision Problems for Computations on Client-server Systems.* Proc. CS&P 2010, pp. 117-125, 2010.